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Turbulent film condensation on a horizontal tube with external flow of pure vapors

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Abstract—The problem of condensation of vapors flowing with high velocity around a horizontal tube is solved for the case of isothermal condition of the wall of a condenser tube. The interfacial shear at the vapor condensate film is assessed with the help of Colburn analogy. Extension of these results in the estimation of average condensation heat transfer coefficients is found to be successful. The present theoretical results are further compared with the recent experimental data and the agreement between the two is found to be very satisfactory. © 1997 Elsevier Science Ltd.

INTRODUCTION

Condensation of pure vapors flowing around a horizontal tube was first tackled by Gomeluri [1]. An asymptotic expression for the estimation of the interfacial shear was employed by them. Their analysis could correlate experimental data for low velocities of the vapor but it was not responding favourably for high velocities of the vapor. Subsequently, many investigations [2–8] concerning the condensation of pure vapors on horizontal tubes can be found in the contemporary literature. In all these analyses the condensate film is assumed to be under laminar flow regime. Further, some treated the wall of the condenser tube to be under isothermal conditions and others considered diabatic conditions of constant heat flux. None of these investigations could correlate the experimental data successfully for high velocities of the vapor flowing around the tube. For example, Michael *et al.* [7] suggested that under high velocities of flow of the vapor, the condensate film can be under turbulent regime. Hence, the present theoretical investigation undertakes an aspect which remained unexplored namely, turbulent film condensation of vapors on a horizontal tube with external flow of vapors.

PHYSICAL MODEL

Pure vapors flow around a horizontal tube such that the flow direction coincides with direction of gravity vector. Further, condensation of vapors taking place on the periphery of the condenser tube is subjected to interfacial shear stress due to the external flow of vapors. The motion of the condensate film is governed by the external forces namely, wall resistance, the body force and the shear force due to the external flow of vapors. The wall of the condenser tube is maintained at a temperature less than the saturation temperature corresponding to the system pressure to facilitate condensation. It is assumed that the condensate film flow is under turbulent regime in the region away from the upper stagnation point. Influence of vapor boundary layer separation of vapors flowing around the cylinder on condensate film is neglected. For the conditions stated and the configuration shown in Fig. 1 the force balance for an elemental film of the condensate can be written as follows:

$$\tau_w = g\delta(\rho_l - \rho_v)\sin\theta + \tau_{iv} \quad (1)$$

where τ_{iv} and τ_w are, respectively, interfacial and wall shear resistances. In equation (1) inertial force is neglected, since the condensate film flow is under turbulent flow regime as per the present model. It is further assumed that turbulent conduction across the condensate layer is more significant than the convective component.

Thus, the energy equation can be written as

$$\frac{d}{dy}\left[\left(1+\frac{\varepsilon_m}{\nu_l}Pr\right)\frac{dT}{dy}\right] = 0. \quad (2)$$

NOMENCLATURE

a	constant in equation (19)
C_p	specific heat capacity of the condensate [J kg^{-1} K^{-1}]
f	average friction coefficient
Fr	Froude number, u_∞^2/gR
Gr	Grashof number, gR^3/ν_l^2
g	acceleration due to gravity [m s^{-2}]
h_{fg}	latent heat [J kg^{-1}]
h	condensation heat transfer coefficient [W m^{-2} °C^{-1}]
k	thermal conductivity [W (mk)$^{-1}$]
Nu	Nusselt number
Nu_m	mean Nusselt number
Pr	Prandtl number
R	outer radius of the tube [m]
R^+	shear radius
R^*	$R^+/Gr^{1/3}$
Re_v	vapor Reynolds number
Re_l	liquid Reynolds number
S	sub-cooling parameter $C_p(T_s-T_w)/(h_{fg}Pr)$
T	temperature [°C]
T^+	dimensionless temperature
u_∞	approach velocity of steam [m s^{-1}]
u^*	shear velocity [m s^{-1}]
u^+	dimensionless velocity (u/u^*)
y	distance normal to the wall [m]
y^+	dimensionless distance (yu^*/ν_L).

Greek symbols

δ	condensate film thickness
δ^+	dimensionless film thickness $(\delta u^*/\nu_L)$
ϕ	shear parameter
μ	absolute viscosity [kg (ms)$^{-1}$]
ρ	density [kg m^{-3}]
τ	shear stress [N m^{-2}]
ν	kinematic viscosity [m^2 s^{-1}].

Subscripts

l	laminar condensate film
v	vapor
iv	vapor–liquid interface
t	turbulent condensate film
w	wall.

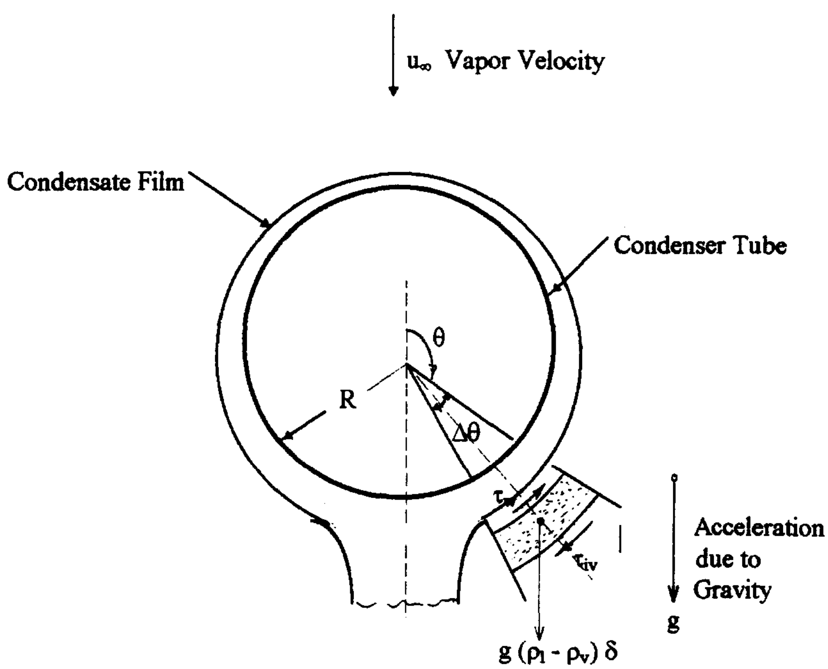

Fig. 1. Physical model and co-ordinate system.

The boundary conditions are as follows:

$$\left.\begin{aligned} \text{at} \quad y = 0; \quad T = T_w \\ y = \delta; \quad T = T_s \end{aligned}\right| . \tag{3}$$

The process of condensation is further represented by the following expression:

$$\frac{d}{d\theta}\int_0^\delta \rho_l u \, dy = \frac{k_l}{h_{fg}} R \frac{dT}{dy}_{y=0}. \tag{4}$$

Equation (4) represents the assumption that all the heat conducted across the film is a result of the condensation of the vapors at the interface.

Table 1. Values of C and n in equation (5)

S.no.	Re	C	n
1	0.4–4	0.989	0.33
2	4–40	0.911	0.385
3	40–4000	0.683	0.466
4	4000–40 000	0.193	0.618
5	40 000–400 000	0.0266	0.805

The interfacial shear resistance 'τ_{iv}' is estimated on the assumption that its order of magnitude would be same as that which would exist for external flow of vapors around cylinders without condensation process occurring on the surface of the cylinder. The asymptotic value of interfacial shear is obtained on the assumption that Colburn analogy holds good for convective heat transfer correlations for cylinders with external flow.

The expression for convective heat transfer around cylinders is as follows, Holman [9]

$$Nu = CRe_v^n Pr^{1/3} \tag{5}$$

where C and n are the entries as shown in the Table 1. Thus, from Colburn's analogy

$$j = f/2 \tag{6}$$

where

$$j = \left[\frac{Nu}{RePr}\right] Pr^{2/3}.$$

One can obtain the mean friction coefficient f as follows

$$f = 2CRe_v^{n-1}. \tag{7}$$

Further, the local value of friction coefficient is defined as

$$f_\theta = C\pi Re_v^{n-1}\sin\theta \tag{8}$$

such that the average friction coefficient 'f' is given by the expression

$$f = \frac{1}{\pi}\int_0^\pi f_\theta \, d\theta. \tag{9}$$

The local shear stress is defined by the relationship

$$\tau_w = \frac{1}{2}\rho_v u_T^2 f_\theta \tag{10}$$

where u_T, the tangential velocity is given by

$$u_T = 2u_\infty \sin\theta.$$

Introducing equation (8) in equation (10) the local shear stress is obtained

$$\tau_{iv} = 2\pi C\rho_v u_\infty^2 Re_v^{n-1}\sin^3\theta \tag{11}$$

where

$$Re_v = u_\infty D/\nu_v.$$

It is evident that in the analysis, boundary layer separation is excluded and in that respect it is based on the assumption employed by Gomeluri and Sheikarladze [1]. Equations (1), (2) and (4) are rendered, respectively, into dimensionless forms with the aid of equation (11) as follows

$$R^{*3} = R^*\phi Fr^{(n+1)/2}\sin^3\theta + \delta^+ \sin\theta \tag{12}$$

$$\frac{d}{dy^+}\left[\left(1+\frac{\varepsilon_m}{\nu_l}Pr\right)\frac{dT^+}{dy^+}\right] = 0 \tag{13}$$

$$\frac{d}{d\theta}\int_0^{\delta^+} u^+\,dy^+ = \left\{\frac{C_p(T_s-T_w)}{h_{fg}Pr}\right\} Gr^{1/3} R^* \frac{dT^+}{dy^+}\bigg|_{y^+=0}. \tag{14}$$

The dimensionless boundary conditions for solving equation (13) are as follows

$$\left.\begin{aligned} T^+ &= 0 \quad \text{at } y^+ = 0 \\ T^+ &= 1 \quad \text{at } y^+ = \delta^+ \end{aligned}\right|. \tag{15}$$

The various dimensionless terms appearing in equations (12)–(14) are as follows:
wall shear parameter

$$R^* = \left(\frac{R^+}{Gr^{1/3}}\right)$$

shear Reynolds

$$R^+ = \left(\frac{Ru^*}{\nu_l}\right)$$

interfacial shear parameter

$$\phi = \left\{2^n \pi C\left(\frac{\rho_v}{\rho_l}\right)\left(\frac{\nu_l}{\nu_v}\right)^{(n-1)} Gr^{(3n-1)/6}\right\}$$

dimensionless temperature

$$T^+ = \left(\frac{T-T_w}{T_S-T_w}\right)$$

modified Grashof number

$$Gr = \left(\frac{gR^3}{\nu_l^2}\right)\left(\frac{\rho_l-\rho_v}{\rho_l}\right)$$

sub-cooling parameter

$$S = C_p(T_s-T_w)/(h_{fg}Pr)$$

Froude number

$$Fr = \left(\frac{u_\infty^2}{gR}\right).$$

Other than these governing dimensionless groupings the rest are given in the Nomenclature. The index 'n' in the estimation of the interfacial shear parameter is to be chosen from Table 1 for the range of Reynolds number encountered in the practice. The local condensation heat transfer coefficients are defined as follows:

$$K_1 \left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T_s - T_w). \tag{16}$$

Equation (16) in dimensionless form can be written as follows:

$$Nu = R^+ \left.\frac{dT^+}{dy^+}\right|_{y^+=0} \tag{17a}$$

or

$$\frac{Nu}{Re_l^{1/2}} = \left.\frac{dT^+}{dy^+}\right|_{y^+=0} \frac{Gr^{1/12}}{Fr^{1/4}} \tag{17b}$$

where

$$Nu = \frac{hR}{k_l}; Re_l = \frac{u_\infty R}{\nu_l}.$$

Further, the mean Nusselt number is obtained from the expression:

$$Nu_m = \frac{1}{\pi}\int_0^\pi Nu\, d\theta. \tag{18}$$

In the present study the Kato's expression for eddy diffusivity is used as given below

$$\frac{\varepsilon_m}{\nu} = 0.4y^+[1-\exp(-ay^{+2})]. \tag{19}$$

Equation (19) is employed by Kato et al. [10] in the investigation of turbulent free convection. Further, Rao et al. [11] made use of equation (19) in the study of condensation of vapors on turbulent falling films. Further, equation (14) requires the knowledge of the velocity profile in the liquid film and it is obtained from the following expression:

$$\frac{du^+}{dy^+} = \frac{1}{\left(1+\frac{\varepsilon_m}{\nu}\right)}. \tag{20}$$

The boundary condition for solving equation (20) is

$$u^+ = 0 \quad \text{at } y^+ = 0. \tag{21}$$

In writing down equation (20), there is an implied assumption that τ_{iv} is of the same order as τ_w. It is presumed that such an assumption will not lead to substantial error in the estimation of the integral appearing in equation (14). In other words universal velocity distribution is being used in the estimation of discharge rate of the condensate at any angular location. In fact, it can be seen that at very high velocities of the external flow of vapor such an assumption, i.e. $\tau_{iv} \to \tau_w$ is valid and hence equation (20) can be considered as a valid approximation.

Thus, the formulation reveals that the local value of the Nusselt is a function of the following dimensionless groups.

$$\frac{Nu}{Re_l^{1/2}} = F\left\{\theta, \phi, Fr, Gr, \frac{C_p\Delta T}{h_{fg}}\frac{1}{Pr}\right\} \tag{22}$$

In order to obtain the functional relationship between the various parameters indicated in equation (22), Equations (12)–(14) and (17) are solved simultaneously as per the scheme outlined.

NUMERICAL METHOD

(1) For the prescribed conditions of the system, the following dimensionless parameters are computed S, Fr, ϕ, Gr.

(2) The numerical procedure is commenced from the node $J = 1$ where $\theta = 0$ and $\delta^+ = 0$. The input parameters are fixed for the given system conditions.

(3) By marching ahead to the next node $J = J+1$, the condensate film thickness $\delta^+(J+1) = \delta^+(J)+\Delta\delta^+$, where $\Delta\delta^+ = 0.3$ is assumed. Equation (14) is written in finite difference form as follows in the evaluation of the angular position 'θ'.

$$\Delta\theta = \frac{I(\delta^+ + \Delta\delta^+) - I(\delta^+)}{SGr^{1/3}R^* \left.\frac{dT^+}{dy^+}\right|_{y^+=0}} \tag{23}$$

where

$$I(\delta) = \int_0^{\delta^+} u^+ dy^+.$$

In the estimation $\Delta\theta$ and R^*, equations (12) and (23) are used iteratively within the prescribed limits of accuracy. When once these values are determined, the value of δ^+ is increased further by $\Delta\delta^+$ and step (2) is repeated in the estimation of the angular position and R^*. Equation (17) is solved to obtain the local Nusselt number. The procedure is repeated till $\theta \to 180$. Further, with the help of equation (18) the average Nusselt number is completed from the local magnitudes of the Nusselt number.

RESULTS AND DISCUSSION

The variation of the condensate film thickness δ^+ from the upper stagnation point to the lower stagnation point is shown plotted in Fig. 2. The variation of the film thickness is found to be monotonically increasing. The dimensionless film thickness δ^+ starts from a zero value since by definition it contains u^*, which assumes zero magnitude at $\theta = 0$ at the upper stagnation point. In addition, when the external flow velocity is increased, the value of the condensate film thickness is increasing. Increase in the value of the sub-cooling parameters 'S' will lead to enhancement of the condensate film thickness at any given angular position. Equation (17) is shown plotted in Fig. 3 and evidently the rise and decay characteristics of the local Nusselt indicates that the condensate heat transfer coefficients reaches a maximum value at $\theta = 90°$ where

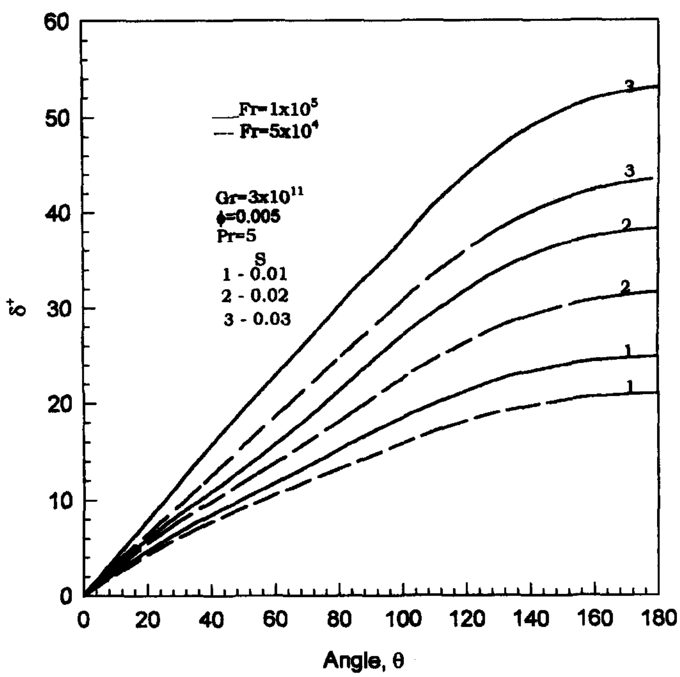

Fig. 2. Variation of condensate film thickness around periphery.

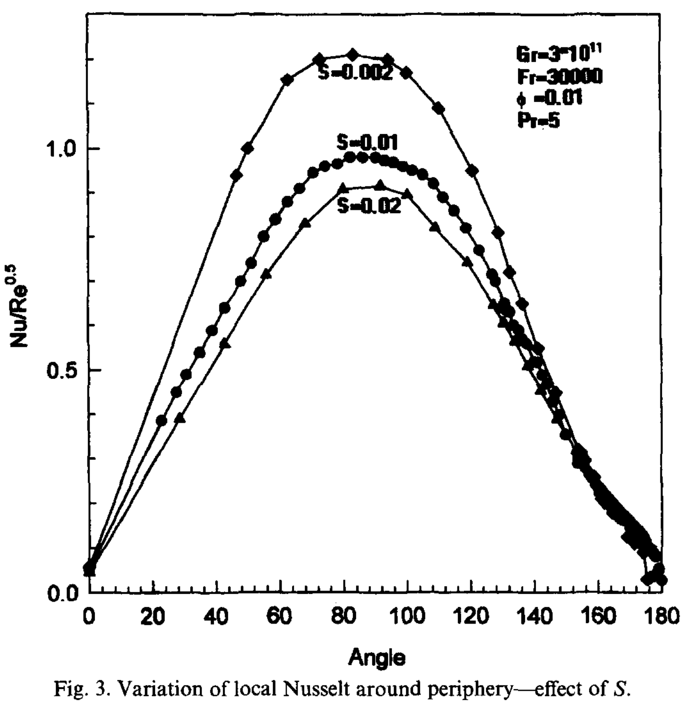

Fig. 3. Variation of local Nusselt around periphery—effect of S.

the gravity effect on the dynamics of the condensate film is maximum for the case of laminar condensation. Further, even for the case of turbulent film condensation, the shear Reynolds also assumes a maximum at $\theta = \pi/2$ and hence Nusselt number is also maximum [see equation (12)]. Further, as $\theta \to \pi$ the condensation heat transfer coefficient decreases considerably. In Fig. 4 the average Nusselt is shown plotted as a function of the sub-cooling parameter 'S' for different values of the external flow velocity of the vapor. It can be observed that the average Nusselt decreases with increase in S. However, for a given value of S, increase in Froude will lead to increase in the average Nusselt. The influence of the shear

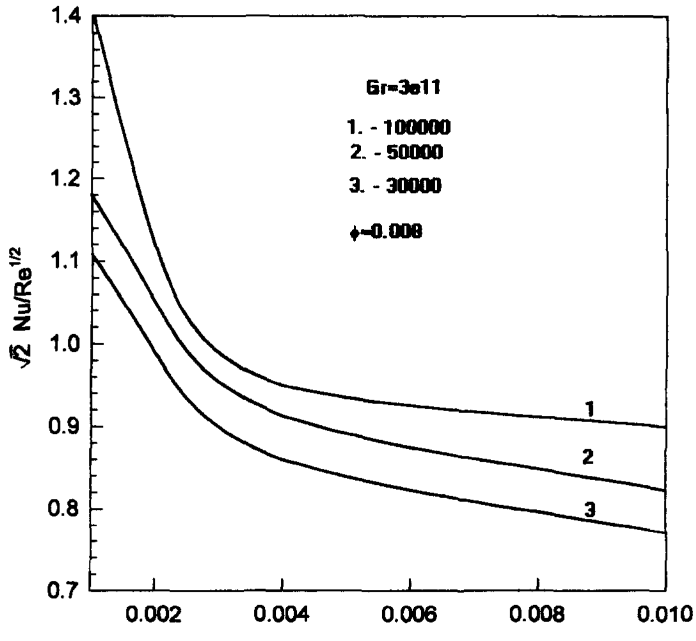

Fig. 4. Effect of sub-cooling parameter on average heat transfer coefficient.

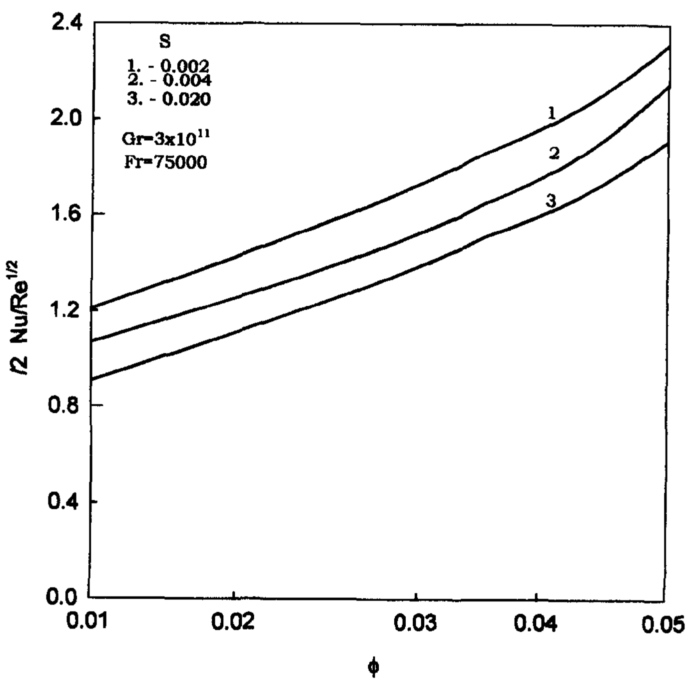

Fig. 5. Effect of interfacial parameter on average Nusselt number—influence of S sub-cooling parameter.

parameter ϕ on the average condensation heat transfer coefficient is shown plotted in Fig. 5. It can be seen that for given system conditions such as the diameter of the tube and external flow velocity, ϕ is dependent on the physical properties such as

$$\left(\frac{\rho_v}{\rho_l}\right) \text{ and } \left(\frac{\nu_l}{\nu_v}\right)$$

which vary with the operating pressure of the condenser. It is observed that as the pressure increases ϕ also increases.

Thus, it can be inferred from Fig. 5 that as the system pressure increases the mean condensation heat transfer coefficient increases. For thicker films of the condensate the average heat transfer coefficient decreases. Figure 6 reveals that as Fr increases at a

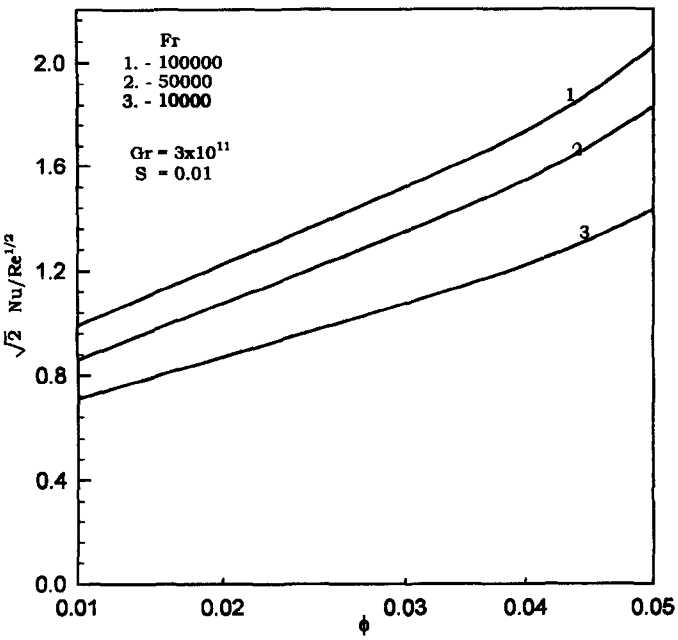

Fig. 6. Effect of Froude on average condensate heat transfer coefficient.

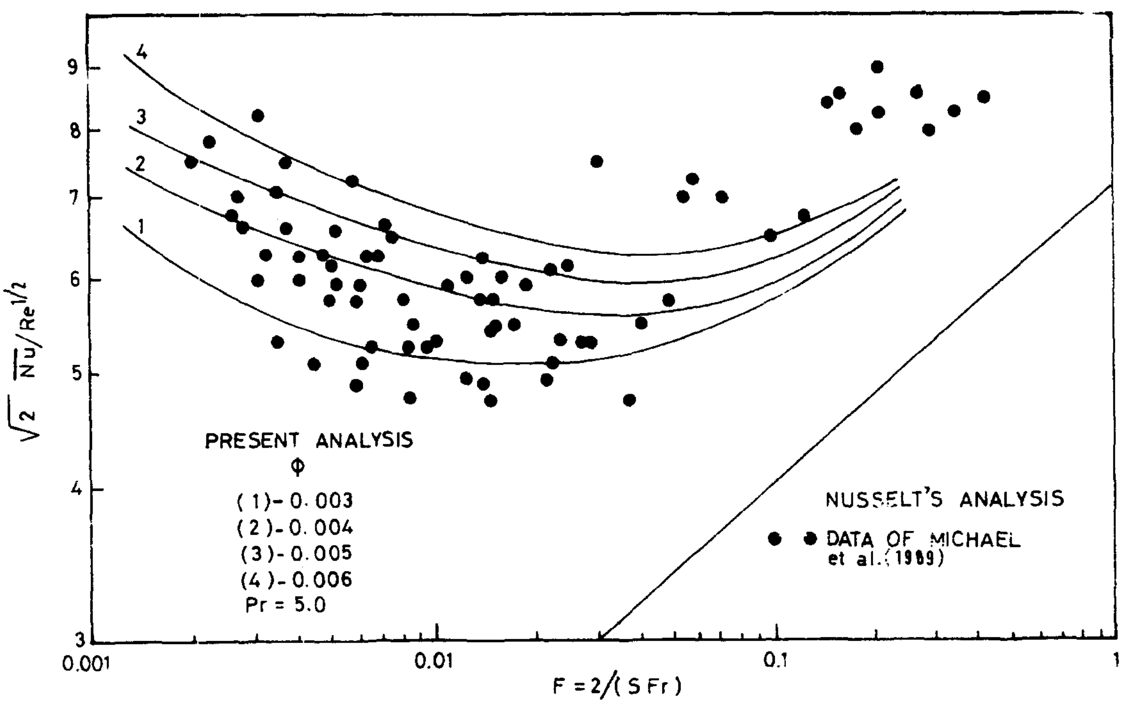

Fig. 7. Comparison of present theory with experimental data.

given system pressure or ϕ = constant the average condensation heat transfer coefficient increases. The theory presented herein is further validated with the recent experimental data of Michael *et al.* [7]. For the possible ranges of the parameters in Fig. 7 the predictions from the present theory are shown plotted together with the experimental data of Michael *et al.* [7]. Evidently, the theoretical analysis predicts satisfactorily the average heat transfer coefficients as can be seen from Fig. 7.

Limiting solution

Further, an attempt is made to develop an equation for the prediction of condensation heat transfer coefficients. For high velocities of the vapor the body force term due to gravity can be detected in equation (12).

Hence, equation (12) can be written as

$$R^{*2} = \phi Fr^{(n+1)/2} \sin^3\theta. \tag{24}$$

Substitution of equation (24) in equation (17) yields

$$\frac{Nu_t}{Re_l^{1/2}} = \phi^{1/2} Gr^{1/12} Fr^{n/4} \left.\frac{dT^+}{dy^+}\right|_{y^+=0} \sin^{3/2}\theta. \tag{25}$$

The average Nusselt number "$Nu_{m,t}$" is given by

$$\frac{Nu_{m,t}}{Re_l^{1/2}} = \phi^{1/2} \frac{Gr^{1/12}}{\pi} Fr^{n/4} I \tag{26}$$

where

$$I = \int_0^\pi \left.\frac{dT^+}{dy^+}\right|_{y^+=0} \sin^{3/2}\theta \, d\theta.$$

The integral I is evaluated and further with the aid of regression for the following ranges of parameters of the present theory, it is represented by the relationship.

$0.002 < \phi < 0.9; 0.002 < S < 0.02$

$$I = 0.0372 S^{-0.131} \phi^{-0.05}. \tag{27}$$

Substitution of equation (27) in equation (26) yields

$$\frac{Nu_{m,t}}{Re_l^{1/2}} = 0.0372 \frac{Gr^{1/2} \phi^{0.45}}{S^{0.3312} F^{0.201}} \tag{28}$$

where

$$F = \left(\frac{2}{FrS}\right).$$

Equation (28) is derived for the case when $4 \times 10^3 < Re_v < 4 \times 10^5$, i.e. $C = 0.0266$ and $n = 0.806$ (see Table).

Further, when the velocity of the vapor is negligible Nusselt's analysis holds good for laminar condensate films. The average heat transfer coefficient is determined from the equation:

$$\frac{Nu_l}{Re^{0.5}} = 0.513 F^{0.25}. \tag{29}$$

Thus, from equations (28) and (29) one can predict for different ranges of the vapor velocity the mean condensation heat transfer coefficients from the relationship:

$$Nu_m^3 = [Nu_l^3 + Nu_{m,t}^3]. \tag{30}$$

Honda *et al.* [4] suggested the type of the functional relationship given in equation (30). Equation (30) of the present analysis, is shown plotted together with the experimental data for R-113. It can be seen that the explicit form of equation (30) satisfactorily correlates the experimental data with reasonable accuracy as can be seen in Fig. 8. Hence, equation (30) can be used for the design purposes.

CONCLUSION

From the theory developed, the following salient conclusions can be arrived at:

(1) It is found that for high velocities of vapors condensing on the condenser tube, estimation of the interfacial shear at the interface by applying Colburn's analogy proved to be successful.

(2) The theory developed herein is found to be in

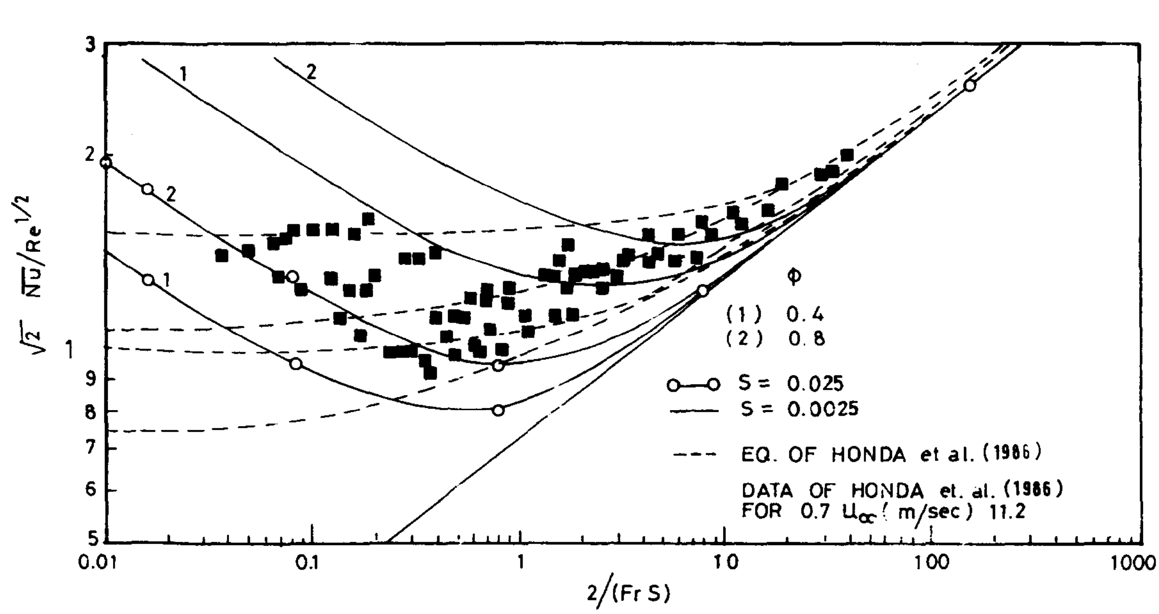

Fig. 8. Comparison of the present analysis with the data Honda *et al.* [4].

good agree with the experimental data of condensation of steam flowing under high velocities and for freon-112.

(3) A limiting solution in explicit form is obtained for high velocities of the vapor. Equation (30) is found to agree satisfactorily for condensation of vapors of R-113 of Honda *et al.* [4]. Hence, equation (30) can be employed for design purposes. Application of Kato's expression of eddy diffusivity is found to yield successful approach in the study of turbulent condensate films on a tube with external flow of vapors.

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